The usefulness of ecological inference for the estimation of voter transition rates

Ioannis Andreadis
Department of Political Sciences
Aristotle University Thessaloniki

Abstract

Although in US based political science journals there are many examples of published papers which use ecological inference methods, it seems that for the European political science journals the use of ecological inference is faced with skepticism. This paper indicates that using ecological inference for the estimation of voter transition rates in the European countries can be a promising path. Simulation results presented in this paper indicate that the total fraction of the electorate misclassified by the used ecological inference algorithm ranges from 1.5% up to 13% with a mean value of about 6%. These figures show that ecological inference could and should be used for the estimation of voter transition rates.

Introduction

During the Elections, Public Opinion and Parties 2008 Annual Conference, I had a discussion with another participant who told me that a paper he had submitted to a European political science journal was rejected, because his findings were based on ecological inference methods. On the contrary, in US based political science journals there are many examples of published papers which use ecological inference methods. (King, Rosen, Tanner and Wagner, 2008; Coan and Holman, 2008; Imai and King, 2004).

This paper is an attempt to change some of the perceptions about ecological inference. Most of these perceptions are based on Robinson's famous paper (Robinson, 1950) which serves as the first example of "ecological fallacy". The output of ecological inference methods, as with every other inference method, strongly depends on the way these methods are applied. The frequently quoted: "garbage in – garbage out" applies here too.

In order to show that ecological inference can be used as a method for the estimation of voter transition rates, this paper includes findings from tests on the validity of these estimates. Since the actual voter transition rates are not known, the assessment of the estimates is based on their closeness to "real" values provided by the simulation of electoral procedures. The paper includes the assessment of both general (for the entire country) and regional estimates. The assessment indicates that, when applied carefully, ecological inference can provide useful estimates of voter transition rates.

The remaining sections of this paper are as follows: Firstly there is a section dedicated to the methodology used including a description of the ecological inference algorithm used and the simulation settings, followed by a section presenting the simulation results. After the simulation results there is a section about the use of ecological inference for the estimation of voter transition rates in the 27 Member states of the European Union including available data and suggestions for the selection of the electoral units in each case. The paper concludes with suggestions for further research.

Methodology

Ecological inference algorithm - VTR

Andreadis and Chadjipadelis (2008) have presented a comparison of methods for the estimation of voter transition rates in Elections, Public Opinion and Parties 2008 Annual Conference. In their paper they have shown that the average voter transitions estimation error using the Markov Chain Monte Carlo Hierarchical Method, a Multinomial-Dirichlet model, proposed by Rosen et al ((Rosen, Jiang, King and Tanner, 2001)) is about seven times greater than the corresponding error using VTR algorithm (Andreadis and Chadjipadelis, 2009). Thus in this paper VTR is the only method that is being tested for the validity of the provided estimates.

This algorithm VTR consists of five tasks: i) initial data preparation, ii) the selection of the best pair of parties, iii) ecological inference for 2x2 tables, iv) extraction of the estimated quantities of interest and data preparation for the next cycle and v) final calculations. A short outline of the method is described as follows: Matrices X (results of the first election per electoral unit), T (results of the second election per electoral unit) and vector N (electorate per electoral unit) are used as input to the algorithm. Each cycle begins with the selection of the "best pair" of parties (i, j) where party i has run for the first election and party j has run for the second election. "Best pair" choice is essentially based on the correlation coefficients. For this pair of parties a 2x2 ecological inference method is applied to estimate the voter transitions from party i to party j in each electoral unit k. Next, the remaining electorate is calculated (by subtracting the estimated voter transitions) and the algorithm proceeds to the next cycle. For a detailed presentation see Andreadis and Chadjipadelis (2009).

National and Local estimates

Usually the validity of ecological inference voter transition estimates for the entire country can be easily verified by comparing them against exit – poll estimates which are usually available on national level. For instance, Chadjipadelis and Andreadis (2009; 2007; 2004) have provided evidence that their ecological inference estimates of voter transition rates in Cyprus, Greece and France are very close to exit-poll estimates.

On the other hand, it is usually more difficult to check the validity of regional estimates of voter transition rates. Exit-poll samples are usually too small to provide valid regional estimates of voter transition rates. As an example, consider an exit poll sample of 4000 respondents. Let's say that party i which has got 30% in the first election lost 10% of its voters towards another party j in a uniform way across all regions. Suppose that we want to estimate the voter transition rate from party i to party j in region k with population that is 5% of the total electorate. Thus, in our sample we have 1200 voters of party i in the first election. From these voters, 120 have moved to party j in the second election. From this group, 6 people live in region k. Small figures like number 6 in our example make it very difficult for exit-poll analysts to announce regional estimates.

Simulation settings[1]

Andreadis and Chadjipadelis (2008) have used simulated voter transition rates between the first and second round of the French Presidential elections that took place in 2007. They have used a matrix with four rows and three columns. The rows correspond to the top four candidates who have received most of the votes in the first round (i.e. SARKOZY, ROYAL, BAYROU and LE PEN). The three columns correspond to the three possible alternatives each voter had in the second round ((i.e. SARKOZY, ROYAL and "not valid + abstention"). They have used the 96 French departments as the electoral units for their analysis.

As an accuracy index of the estimation of the general transition matrix they have used the formula $\sum_{i=1}^{4}\sum_{j=1}^{3}\left|\hat{p}_{ij}-p_{ij}\right|$ where $\hat{p}_{ij}$ is the estimated national voter transition rate from party i to party j and p_{ij}, is the "real" national transition rate calculated from the simulated values. The absolute deviation calculated by the aforementioned formula represents the total number of voters (as a fraction of the electorate) who were misclassified by the method. In the same paper they have demonstrated that the average absolute deviation of the VTR algorithm national level estimates is 10,89%.

This paper follows a similar approach on an improved version of VTR algorithm. The same 4x3 matrix is used as the template for the simulation and the same absolute deviation index is used to check the accuracy of the estimates. But in addition to presenting results from an improved version of VTR, this paper presents the accuracy index of both national and regional estimates.

The simulation is set in the following way: In each cycle suppose that the real 4x3 national transition matrix is given in Table 1. The values p_{ij} in this table correspond to the number of voter transitions divided by the size of the total electorate.

Table 1 General transition matrix

	SARKOZY	ROYAL	Non Valid + Abstention
SARKOZY	0.3503	0.0036	0.0072
ROYAL	0.0015	0.2901	0.0044
BAYROU	0.0874	0.0852	0.0459
LE PEN	0.0797	0.0174	0.0274

Since the electoral unit used is the French department, for each of the 96 departments the simulation draws a sample from a multinomial distribution Multinomial(n_k, $\boldsymbol{p^*}$) with 12 possible outcomes, where n_k is the size of the electorate in department k=1, 2, …, 96 and $p^*_{ij} \sim Normal(p_{ij},\ p_{ij}/6)$. Thus, for each department we have n_k independent trials from a multinomial distribution, where each trial results in exactly one of the 12 possible outcomes, with probabilities $\boldsymbol{p^*}$. Also note that in each district the probabilities are different and the only assumption is that if the national transition rate from party i to party j is p_{ij}, then in department k the corresponding transition rate p_{ijk} will follow a Normal distribution with mean value p_{ij} and standard deviation $p_{ij}/6$.

[1] For the code used see the Appendix.

The algorithm stores the sample values to a 4x3x96 array with elements n_{ijk} where i=1, 2, 3, 4, j=1, 2, 3 and k=1, 2, …, 96. The values of n_{ijk} represent the number of voters who have moved from party i to party j in department k. Summing for all k we get the 4x3 matrix with elements n_{ij}, where n_{ij} is the number of voters who have moved from party i to party j in all departments. Summing for all j we get the 96x4 matrix X with elements $n_{i\cdot k}$, where $n_{i\cdot k}$ is the number of voters of the first round of the elections who have voted for party i in department k. Summing for all i we get the 96x3 matrix T with elements $n_{\cdot jk}$, where $n_{\cdot jk}$ is the number of voters of the second round of the election who have voted for party j in department k. Then matrices X and T along with vector $N=(n1, n2, \ldots, n_k)$ are used as input to VTR algorithm. The aforementioned procedure was replicated for 100 times and the findings are presented in the following section.

Simulation Results

Table 2 includes summary statistics of absolute deviations for VTR algorithm after 100 replicates of the simulation procedure. National level estimates (first row of the table) are very close to simulated values. The average total number of voters (as a fraction of the electorate) who were misclassified by VTR is 6.18%. The maximum absolute deviation of the voter transition rates estimates from the simulated (true) values is 13.01%. For half of the cycles the absolute deviation is in the range (4.63%, 7.37%)

Table 2 Summary statistics of absolute deviations

Estimates	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
National	1.56%	4.63%	6.16%	6.18%	7.37%	13.01%
Regional	0.67%	4.73%	6.37%	6.52%	7.94%	16.37%
Department	0.88%	5.17%	6.94%	7.30%	9.08%	22.71%

Estimating department level voter transition rates using as electoral units the same departments is a more difficult task. The average percentage of the electorate misclassified by VTR is 7.30% on the department level. The maximum absolute deviation of the voter transition rates estimates from the simulated (true) values on the department level is 22.71%.

The selection of electoral units is a very important factor for the successful estimations of quantities of interest. Andreadis and Chadjipadelis (2009) argue that as electoral units we should use the most detailed level of available electoral outcomes for which the electorate has remained unchanged between the first and the second election. Even if we are not interested in presenting our results on such a detailed level we should use this level and then combine the corresponding estimates and use the weighted means of the low level estimates to produce the estimates for the level we want to present. This approach offers the advantage of having more available data that will provide more deterministic information for the feasible range of values of quantities of interest.

Thus, according to Andreadis and Chadjipadelis, if we want to produce estimates of voter transition rates for regions defined by a division of the electorate, it is better to use electoral units defined by a subdivision of this division. For instance, in France if we want to estimate voter transition rates for the 22 Regions, it is better to use the 96

Departments as electoral units in VTR. The benefit is obvious from the second row of Table 2. The average percentage of the electorate misclassified by VTR is 6.37% on the regional level. The maximum absolute deviation of the voter transition rates estimates from the simulated (true) values on the regional level is 16.37%.

The maximum absolute deviation is different for each region and it is related with the number of departments included in the regions. In France there are regions with only two departments and other regions with up to eight departments. The maximum absolute deviation presented in Table 2 is among all regions. Restricting the searching of maximum only to the regions of France which have eight departments the corresponding value is 14.24%.

It should be noted that the absolute deviation index presented so far is the sum of absolute deviations for all 12 cells of the transition matrix. Another useful index is the average absolute deviation which is calculated by the formula: $\frac{1}{12}\sum_{i=1}^{4}\sum_{j=1}^{3}\left|\hat{p}_{ij} - p_{ij}\right|$. This index shows the absolute average deviation of the estimated value from the simulated value in each of the 12 cells of the general transition matrix. This index is included because it is helpful to remember that a 6% total absolute deviation in our case with a 4x3 transition matrix means that the value of average absolute deviation is 0.5%, i.e. for the average cell the difference between the estimates and the real values is 0.5%.

Available data in EU

Estimation of voter transition rates with ecological inference methods is done using electoral results from two consecutive elections. Thus, it is useful to know what the available data are for each of the 27 member states of the European Union. The numbers of first and second level administrative divisions for which there are available electoral results are shown in Table 3 for each European Union country. According to the elements of this table the average unit of the first level electoral division of a European country consists of about 21 second level electoral division units. Spain, with 17 autonomous communities and 52 provinces, is the country which according to this table presents the smaller number of second level units per first level unit (3.1). On the other hand, UK presents the largest number with an average of 161.5 constituencies per country.

Table 3. Availability of electoral results

Member	1st	Level name	2nd	Level name	Average
Austria	9	States	121	Districts[2]	13.4
Belgium	11	Constituencies	208	Cantons	18.9
Bulgaria	28	Provinces	260	Municipalities	9.3
Cyprus	5	Districts	334	Municipalities	66.8
Czech Republic	14	Regions	91	Territorial units	6.5
Denmark	5	Regions	98	Municipalities	19.6
Estonia	15	Counties	261	Municipalities	17.4
Finland	15	Electoral districts	416	Municipalities	27.7
France[3]	22	Regions	96	Departments	4.4

[2] In Austria there are 98 (Politische Bezirke and Statutarstädte) and Vienna which is divided into 23 districts

Germany	16	States	299	Districts	18.7
Greece	56	Electoral districts	1034	Municipalities	18.5
Hungary	20	Regions	173	Sub regions	8.7
Ireland	43	Constituencies	192	Local Electoral Areas	4.5
Italy	27	Districts	110	Counties	4.1
Latvia	5	Districts	34	Regions	6.8
Lithuania	71	Constituencies	2034	Districts	28.6
Luxembourg	12	Cantons	118	Municipalities	9.8
Malta	13	Districts	55	Localities	4.2
Netherlands	12	Provinces	465	Municipalities	38.8
Poland	16	Provinces	379	Counties	23.7
Portugal	20	Districts	308	Municipalities	15.4
Romania	42	Counties	312	Colleges	7.4
Slovakia	8	Regions	79	Districts	9.9
Slovenia	8	Constituencies	88	Districts	11.0
Spain	17	Autonomous communities	52	Provinces	3.1
Sweden	21	Counties	290	Municipal assemblies	13.8
United Kingdom	4	Countries	646	Constituencies	161.5

Electoral units for national estimates

The simulation results using as template the first and second level administrative (and electoral) French divisions indicate that the number of units is a significant factor for the accuracy of estimates of voter transition rates. Considering this, it seems that to produce national level estimates of voter transition rates in Romania, Ireland, Greece and Lithuania, (which all are divided to more than 40 areas using their first level division) we can use as electoral units the areas defined by the first level division. As the number of areas gets smaller we can be less confident for the estimates. For instance, it would not be wise to use the 5 districts of Cyprus or Latvia, the 5 regions of Denmark, or the 4 countries of United Kingdom to produce general estimates.

For the rest of the member states, researchers have to decide considering the level of accuracy they require and the special characteristics of each country. For instance, in Spain there are 17 autonomous communities and a lot of local political parties which run for the elections only in selected areas. An analysis on Spanish elections would be better if the researcher would use the 52 provinces or even a more detailed division. In fact, some preliminary results on the application of VTR in Spain indicate that it is better to apply the algorithm in each autonomous community separately. This could be also true for other member states of the EU (i.e. federal states comprising a number of partially self-governing regions) and United Kingdom consisting of four countries and having Scottish, Welsh and Northern Ireland parties although VTR can handle similar cases to some degree.

[3] France is divided into 22 regions and four departments which lie overseas, the latter departments are also counted as regions totalling the number of France regions to 26.

Regional estimates

Apart from Spain and UK which were discussed above, according to the elements of Table 3, in the rest of the EU Members the average unit of the first level electoral division includes a number of second level electoral division units which number ranges from about 4 (Italy, Malta, France) up to more than 66 (Cyprus). As the simulation results indicate when the ratio between first and second level divisions is about 1 to 4 and the second level division units are used as electoral units, the average percentage of the electorate misclassified by VTR on the fist division level is 6.37% and the corresponding maximum value is 16.37%.

Thus, it seems that the interested readers could use ecological inference for the estimation of voter transition rates on both national and regional levels in all the 27 European Union countries.

Conclusions

In the aftermath of an election, political parties need to analyse their performance at the maximum possible detailed level. If they have raised their power, they need to know previous electoral behaviour of their new voters. If they have suffered from defections, they need to know the political parties which have earned from these defections. Ecological inference can serve as a useful tool for these cases because it provides a general table of voter transitions for the entire country and a detailed array of voter transitions for each of the departments included in this country.

The simulation findings presented in this paper indicate that the VTR algorithm provides estimates which are close to reality and it suggests that the method could and should be used in all the member states of the European Union. This paper includes preliminary results on the validity of ecological inference for the estimation of voter transition rates on both national and regional levels. Further research should include more extensive simulations, i.e. with more than 100 replicates/cycles and with larger than 4x3 tables. A full proof of the usefulness of ecological inference for the estimation of voter transition rates for all the 27 European Union Members could be given by two ways. The first one would be by using a more general simulation experiment with random selection of rows and columns and random selection of the general voter transition matrix. The second way would be to run a separate simulation experiment for each one of the 27 European Union Members using a national specific general transition matrix for each one of them.

References

Andreadis, I and Chadjipadelis, T. (2009) 'A method for the estimation of voter transition rates', *Journal of Elections, Public Opinion and Parties,* 2 .

Andreadis, I and Chadjipadelis, T. (2008) 'Comparison of methods for the estimation of voter transition rates', *Elections, Public Opinion and Parties 2008 Annual Conference*, Manchester, UK.

Chadjipadelis, T. and Andreadis, I. (2007) 'Analysis of the Cyprus Referendum on the Annan Plan', *57th Political Studies Association Annual Conference 'Europe and Global Politics'*, Bath, UK.

Chadjipadelis, T. and Andreadis, I. (2004) 'Electoral analysis. The case of Popular Orthodox Rally. Profile of its voters. (Greek)' in C. Vernardakis, H. Georgantas, D.

Gravaris and D. Kotrogiannos (eds). *Thirty years of democracy: The political system of the third Hellenic Republic 1974-2004.* KRITIKH, pp. 176-195.
Coan, T. G. and Holman, M. R. (2008) 'Voting Green*', *Social Science Quarterly,* 89 (5), 1121-1135.
Imai, K. and King, G. (2004) 'Did illegal overseas absentee Ballots decide the 2000 US presidential election?', *Perspectives on Politics,* 2 (03), 537-549.
King, G., Rosen, O., Tanner, M. and Wagner, A. F. (2008) 'Ordinary Economic Voting Behavior in the Extraordinary Election of Adolf Hitler', *The Journal of Economic History,* 68 (04), 951-996.
Robinson, W. S. (1950) 'Ecological Correlations and the Behavior of Individuals', *American Sociological Review,* 15 (3), 351-357.
Rosen, O., Jiang, W., King, G. and Tanner, M. A. (2001) 'Bayesian and Frequentist Inference for Ecological Inference: The R × C Case', *Statistica Neerlandica,* 55 134-156.

Appendix Simulation code in R language

```
source("VTR.R") #load VTR algorithm (download from
http://polres.polsci.auth.gr)

# Initialize values which should be changed for each
country
MaxRepl<-100 # number of replications
DimX<-4 # number of parties in e1
DimT<-3 # number of parties in e2
NRegions<-22 # number of regions
NDeps<-96 # number of Departments
nfrance<-read.csv2("nfrance.csv") # file with NDeps rows
and two columns: nk values and region codes
regions<-factor(nfrance$reg)
p<-c(0.3503,0.0015,0.0874,0.0797,0.0036,0.2901,0.0852,
0.0174,0.0072,0.0044,0.0459,0.0274) #general voter
transition matrix for France

# Define arrays to store the values
x.ijk<-array(0, dim=c(DimX,DimT,NDeps)) # the number of
people who voted party i in e1 and party j in e2 in
department k
tr.ijk<-array(0, dim=c(DimX,DimT,NDeps)) # transition
rate of voters of party i who voted for party j in
department k
vtr<-array(0, dim=c(DimX,DimT,MaxRepl)) # the estimated
general transition matrix for each replicate (as a
fraction of power of party i)
vtr96<-array(0, dim=c(NDeps,DimX,DimT,MaxRepl)) #
estimated voter transitions as a fraction of power of
party i in department k
vtrtotal96<-array(0, dim=c(NDeps,DimX,DimT,MaxRepl)) #
estimated voter transitions as a fraction of the total
electorate in department k
vtrtotal22<-array(0, dim=c(NRegions,DimX,DimT,MaxRepl)) #
estimated voter transitions as a fraction of the total
electorate in each region
devanal<-array(0, dim=c(DimX,DimT,MaxRepl)) #absolute
deviations from the simulated "true" values
devanal96<-array(0, dim=c(NDeps,DimX,DimT,MaxRepl))
#absolute deviations from the simulated "true" values in
each department
devanal22<-array(0, dim=c(NRegions,DimX,DimT,MaxRepl))
#absolute deviations from the simulated "true" values in
each region
vtrdev<-array(0,MaxRepl) #sum of absolute deviations
vtrdev96<-array(0, dim=c(NDeps,MaxRepl)) #sum of absolute
deviations in each department
vtrdev22<-array(0, dim=c(NRegions,MaxRepl)) #sum of
absolute deviations in each region
```

```
disturb<-6 # define the standard deviation as 1/6 of the
mean value

# Replication starts
for (repl in c(1:MaxRepl)) {
cat(repl)
print(date())
set.seed(repl) # to keep track of the simulated values
and check outliers

#Draw values in each department
for (k in c(1:NDeps)) {
# in each department the voter transition rate follows a
Normal distribution with mean value p and sd p/distrub
# (it needs to be reformed to work with any number of
dimensions)
p2<-c(rnorm(1, mean = p[1], sd = p[1]/disturb),
rnorm(1, mean = p[2], sd = p[2]/disturb),
rnorm(1,mean = p[3], sd = p[3]/disturb),
rnorm(1,mean = p[4], sd = p[4]/disturb),
rnorm(1,mean = p[5], sd = p[5]/disturb),
rnorm(1,mean = p[6], sd = p[6]/disturb),
rnorm(1,mean = p[7], sd = p[7]/disturb),
rnorm(1,mean = p[8], sd = p[8]/disturb),
rnorm(1,mean = p[9], sd = p[9]/disturb),
rnorm(1,mean = p[10], sd = p[10]/disturb),
rnorm(1,mean = p[11], sd = p[11]/disturb),
rnorm(1,mean = p[12], sd = p[12]/disturb))
p2[p2<0]<-0.00001 # if a simulated value is negative put
a small value for the corresponding probability
nk<-nfrance$nk[k] # size of electorate in each department
nij<-rmultinom(n=1,size=nk,prob=p2) # draw from a
multinomial distribution
# (it gives the number of voters who moved from party i
to party j in department k)
dim(nij)<-c(DimX,DimT) # change the dimensions to fit the
dimensions of the voter transition matrix
x.ijk[,,k]<-nij # the simulated "true" number of people
who voted party i in e1 and party j in e2 in department k
tr.ijk[,,k]<-x.ijk[,,k]/rowSums(x.ijk[,,k]) # transition
rate of voters of party i who voted for party j in
department k
}
# Values were drawn for each department

# Prepare data for input in VTR
n.ij <- apply(x.ijk,c(1,2),sum) # general voter
transition matrix (absolute values) (kept for testing)
n.ik <- apply(x.ijk,c(1,3),sum) # number of voters of
party i; matrix with dimensions(DimX, NDeps)
```

```
n.jk <- apply(x.ijk,c(2,3),sum) # number of voters of party j; matrix with dimensions(DimT, NDeps)
n.k <- apply(x.ijk,3,sum) # number of voters in department k; vector with length(NDeps); it should be the same with nk; (kept for testing)
data<-as.data.frame(cbind(n.k,t(n.ik),t(n.jk))) #prepare the data frame (N,X,T)
data[-1]<-data[-1]/n.k # calculate the relative values (vote share for each party in each department)
MyX<-data[2:5]
MyT<-data[6:8]
MyN<-data[1]

# Run VTR
z<-multirate(MyN,MyX,MyT,0.001)
vtr[,,repl]<-z$Bb # store the estimated general transition matrix for the current replicate (as a fraction of power of party i)
vtrtotal<-pij*z$Bb #calculate voter transitions as a fraction of the total electorate
vtr96[,,,repl]<-z$bb # store the estimated transition matrix of each department for the current replicate (as a fraction of power of party i)
pij<-rowSums(n.ik)/sum(n.ik) # calculate the national relative values of parties in X (vote shares)
pij96<-t(n.ik)/n.k # calculate the relative values of parties in X(vote share for each party in each department)
for (j in c(1:DimT)) {
vtrtotal96[,,j,repl]<-pij96*z$bb[,,j] #calculate and store voter transitions as a fraction of the total electorate in department k
}

#Calculate the estimated values for each region
n.22<-aggregate(n.k, list(regions), sum)
vtrtotal22temp<-aggregate(vtrtotal96[,,,repl]*n.k, list(regions), sum)
vtrtotal22temp<-as.matrix(vtrtotal22temp[-1])
dim(vtrtotal22temp)<-c(NRegions,DimX,DimT)
vtrtotal22temp<-vtrtotal22temp/n.22$x
vtrtotal22[,,,repl]<-vtrtotal22temp


#Calculate the simulated "true" voter transition rates as a fraction of the total electorate
theta<-n.ij/sum(n.k) # simulated "true" voter transition rates as a fraction of the total electorate
newx.ijk<-aperm(x.ijk, c(3,1,2))
```

```
theta96<-newx.ijk/n.k # simulated "true" voter transition
rates as a fraction of the total electorate in each
department
theta22<-aggregate(newx.ijk, list(regions), sum)
theta22<-as.matrix(theta22[-1])
dim(theta22)<-c(NRegions,DimX,DimT)
theta22<-theta22/n.22$x # simulated "true" voter
transition rates as a fraction of the total electorate in
each region

devanal[,,repl]<-abs(vtrtotal-theta) #calculate absolute
deviations from the simulated "true" values
vtrdev[repl]<-sum(devanal[,,repl]) #sum of absolute
deviations
devanal96[,,,repl]<-abs(vtrtotal96[,,,repl]-theta96)
#calculate absolute deviations from the simulated "true"
values in each department
vtrdev96[,repl]<-rowSums(devanal96[,,,repl]) #sum of
absolute deviations in each department
devanal22[,,,repl]<-abs(vtrtotal22[,,,repl]-theta22)
#calculate absolute deviations from the simulated "true"
values in each region
vtrdev22[,repl]<-rowSums(devanal22[,,,repl]) #sum of
absolute deviations in each department
}
# End of replication
```